EXTRACTION IN A COLUMN FOR THE CASE OF SOLID SPHERICAL PARTICLES WITH A POLYDISPERSE COMPOSITION

A. É. Golland

UDC 66.061.4

The problem of extraction of a substance in a column is considered for the case of solid spherical particles with the polydisperse composition when mass transfer is controlled by the liquid phase resistance and the distribution coefficient of the extractable substance is constant. For the exponential size-particle distribution we present the results of the corresponding calculation of the extraction degree. It is shown that for the normal distribution the calculation of the extraction degree can be performed without reference to polydispersity of the particles with an accuracy sufficient for engineering computations. The calculations were carried out in the range of values for the volume fraction of the particles from 0 to 0.666.

The problem concerning the calculation of the extraction degree of a substance from a granular material in a column by an absorbing solution when mass transfer is controlled by the liquid phase resistance, and the distribution coefficient of the extractable substance is constant, is analogous to the familiar problem on sorption from a gaseous flow [1]. The corresponding solution to the extraction degree of the extractable substance is well known:

$$X = \exp(-x_1) \int_0^{t_1} I_0 (2 \sqrt{2 x_1 \eta}) \exp(-\eta)\, d\eta. \tag{1}$$

However, it is not clear how much an application of the given solution to the polydisperse material is substantiated for the case when the real size-particle distribution is replaced by mean characteristics. Below we consider the indicated problem for the spherical particles at low Reynolds numbers when mass transfer occurs under the Stokes conditions.

The mathematical description of the extraction process in this situation takes on the following form:

$$\begin{gathered}
\frac{\partial c_0}{\partial x_1} + \gamma \sum_{i=1}^{N} p_i \frac{\partial a_{0i}}{\partial t_1} = 0,\ \gamma \frac{\partial a_{0i}}{\partial t_1} = -\varphi_i (\gamma a_{0i} - c_0),\ a_{0i} = \varepsilon a_i, \\
c_0 = (1-\varepsilon) c,\ x_1 = \beta \frac{x}{v},\ t_1 = \gamma\beta \left(\tau - \frac{x}{v}\right),\ c_0 (0,\ t_1) = 0, \\
c_0 (x_1,\ 0) = 0,\ a_{0i} (x_1,\ 0) = a_{\text{in}} \varepsilon,\ \gamma = \frac{\psi (1-\varepsilon)}{\varepsilon},\ \varphi_i = \frac{\beta_i}{\beta}, \\
\beta_i = \frac{6 \varepsilon \alpha_i}{(1-\varepsilon) d_i},\ \beta = \sum_{i=1}^{N} p_i \beta_i.
\end{gathered} \tag{2}$$

Mass transfer to the solid spherical particles streamlined by a constricted liquid flow with low Reynolds numbers and high Peclet numbers is considered in [2].

Here for the mass transfer coefficient we obtain the equation

$$\begin{gathered}
\alpha_i = 0{,}998 \frac{D}{d_i} (1 + d_i B)^{1/3} \mathrm{Pe}^{1/3}, \\
B = \frac{1}{2 - 3\varepsilon} \left\{ \left[18\varepsilon \left(1 - \frac{3}{2}\varepsilon\right) \frac{b_1}{b_3} + \frac{81}{4} \varepsilon^2 \left(\frac{b_2}{b_3}\right)^2 \right]^{1/2} + \frac{9}{2} \varepsilon \frac{b_2}{b_3} \right\}.
\end{gathered} \tag{3}$$

G. S. Petrov Scientific-Research Institute of Plastics, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 63, No. 3, pp. 350-353, September, 1992. Original article submitted October 22, 1991.

1062-0125/92/6302-0935$12.50

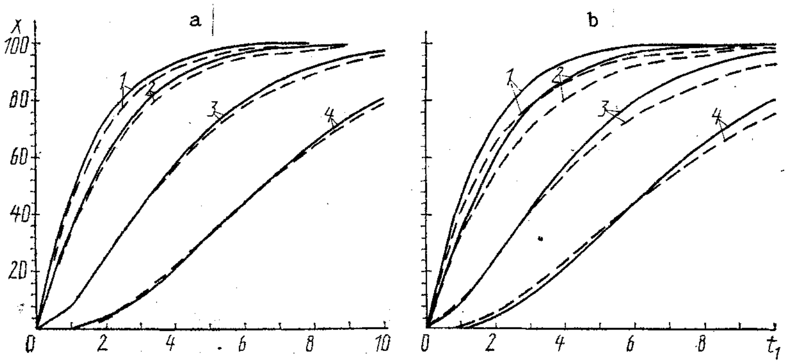

Fig. 1. Dependence of extraction degree X, %, on dimensionless values of coordinate x_1 and time t_1 for $\varepsilon = 0.5$ and the normal (a) and the exponential (b) size-particle distribution laws: 1) $x_1 = 0.5$; 2) 1.0; 3) 3.0; 4) 6.0; solid curves) calculation by Eq. (1); dashed curves) calculation by Eqs. (4), (5).

Then the formula for coefficients φ_i becomes:

$$\varphi_i = \frac{(1+Bd_i)^{1/3}}{d_i^{5/3}} \frac{1}{\sum\limits_{i=1}^{N} p_i \frac{(1+Bd_i)^{1/3}}{d_i^{5/3}}}. \tag{4}$$

The solution to the problem of (2), (4) may be obtained, for example, by means of the Laplace transformation. The extraction degree is expressed in this case as follows:

$$X = \sum_{i=1}^{N} L^{-} \left[\frac{p_i \varphi_i}{s(s+\varphi_i)} \exp\left(-sx_1 \sum_{i=1}^{N} \frac{p_i \varphi_i}{s+\varphi_i} \right) \right]. \tag{5}$$

For i = 1 Eq. (5) is transformed into (1).

The numerical evaluation of the concrete value of X can be comparatively simply obtained by Papoulis's method [4]. It has been shown that the discrepancy between the results of numerical integration by (1) and the numerical handling of the Laplace transformation for expression (5) at i = 1 is less than one percent.

We carried out the calculation of the extraction degree by the relations of (4), (5) for ε from 0 to 0.666 and of two size-particle distribution laws frequently used in practice, namely, the normal and exponential distribution laws. The quantity B takes on the following form:

the normal distribution

$$B = \frac{\sqrt{18\varepsilon(1-3\varepsilon/2)(1+3\sigma^2)+81\varepsilon^2(1+\sigma)^2/4+9\varepsilon(1+\sigma^2)/2}}{(2-3\varepsilon)(1+3\sigma^2)b_1},$$

the exponential distribution

$$B = \frac{\sqrt{3\varepsilon(1-3\varepsilon/2)+9\varepsilon^2/4}+3\varepsilon/2}{(2-3\varepsilon)b_1}.$$

The results obtained were compared with the magnitude of the extraction degree for the monodisperse material, being calculated from (1) with the same values of the dimensionless variables x_1 and t_1.

The examples of the extraction degree calculations for the normal and exponential distributions are presented in Fig. 1. It is shown that for both distributions the dependence of X on x_1 and t_1 is practically independent of ε.

For the normal distribution the error between the calculations by Eqs. (4), (5) and (1) is defined by quantity σ. However, extraction degrees is more than 10% the error is insignificant (less than 5%), therefore, when performing engineering computations such error may be neglected. For example, for the curves presented in Fig. 1a ($\sigma = 0.333$), at $X > 10\%$ the maximum error between the exact and approximate calculation does not exceed 5%, whereas the mean error amounts to 1.5-2.5%.

TABLE 1. Extraction Degree X, %

t_1	x_1							t_1	x_1						
	0,25	0,50	1,00	2,00	4,00	6,00	10,0		15,0	20,0	25,0	30,0	35,0	40,0	45,0
0,50	28,5	23,9	16,7	7,9	1,4	0,0	0,0	10,0	17,0	4,9	0,7	0,0	0,0	0,0	0,0
1,00	47,4	41,7	32,0	18,1	4,8	0,6	0,0	12,5	31,1	11,5	3,6	0,9	0,0	0,0	0,0
1,50	60,9	55,2	44,9	28,7	10,0	2,4	0,0	15,0	46,4	21,4	7,9	2,8	1,1	0,0	0,0
2,00	70,0	64,8	55,1	38,3	16,2	5,4	0,0	17,5	60,7	33,6	14,7	5,5	2,3	1,2	0,4
2,50	77,3	72,6	63,7	47,2	22,8	9,3	0,0	20,0	72,9	46,9	24,2	10,2	3,9	2,0	1,4
3,00	82,6	78,7	70,7	55,0	29,6	13,8	1,0	22,5	82,1	59,6	35,5	17,3	7,0	2,8	1,7
3,50	86,0	82,7	75,6	61,4	36,1	18,6	2,7	25,0	88,3	70,3	47,3	26,4	12,2	4,8	2,1
4,00	88,8	86,1	80,2	67,3	42,5	23,7	4,8	27,5	92,9	79,2	58,6	36,9	19,4	8,5	3,3
4,50	92,0	89,8	84,3	72,7	48,6	28,9	7,2	30,0	96,0	86,0	68,7	47,7	28,3	14,1	5,8
5,00	93,8	91,8	87,3	76,9	54,2	34,1	10,0	32,5	97,8	90,9	77,1	58,0	38,0	21,4	10,1
6,00	96,2	94,8	91,4	83,3	63,8	44,0	16,1	35,0	98,9	94,3	83,8	67,3	48,0	29,9	15,9
7,00	97,2	96,1	93,8	87,9	71,8	53,4	22,9	37,5	99,0	96,3	88,8	75,2	57,4	39,0	23,1
8,00	97,8	97,3	95,7	91,5	78,5	61,9	30,4	40,0	99,5	97,8	92,5	81,8	66,1	48,2	31,3
9,00	99,8	99,2	98,1	94,6	84,0	69,4	38,0	42,5	100,0	98,6	95,1	87,0	73,7	57,0	39,8
10,00	100,0	99,6	98,8	96,3	88,1	75,6	45,6	45,0	100,0	99,5	97,1	91,1	80,2	65,2	48,4
12,00	100,0	100,0	99,8	98,4	93,4	84,9	59,7	47,5	100,0	100,0	98,0	93,9	85,3	72,3	56,6
14,00	100,0	100,0	100,0	99,4	96,8	91,3	71,6	50,0	100,0	100,0	98,7	96,0	89,5	78,6	64,3
16,00	100,0	100,0	100,0	99,5	98,1	94,8	80,6	55,0	100,0	100,0	99,5	98,5	95,2	88,1	77,3
18,00	100,0	100,0	100,0	100,0	98,8	97,1	87,1	60,0	100,0	100,0	99,9	99,6	98,2	94,3	86,8
20,00	100,0	100,0	100,0	100,0	99,1	97,7	91,3	65,0	100,0	100,0	100,0	100,0	99,6	97,8	93,3
30,00	100,0	100,0	100,0	100,0	100,0	100,0	99,6	75,0	100,0	100,0	100,0	100,0	100,0	99,8	99,0

For the exponential distribution the corresponding error between the calculations increases substantially and can reach 20-25% (see Fig. 1b). The exact values of the extraction degree for the polydisperse spherical material with $\varepsilon = 0.333$ and the exponential distribution are given in Table 1.

Thus, for the normal distribution law the calculation of the extraction degree without reference to polydispersity of the spherical particles does not lead to essential errors, while for the exponential distribution law the error may increase considerably.

NOTATION

x, τ, coordinate along the column length and time, respectively; x_1, t_1, dimensionless values of the coordinate along the column length and time; I_0, Bessel function of the imaginary-argument zero order; c, a_i, the extractable substance concentration in a liquid phase and in particles of diameter d_i; $X = (a_{in} - \Sigma_{i=1}^{N} a_i p_i)/a_{in}$, the extraction degree; ε, the volume concentration of particles in a column; φ, the distribution coefficient of the extractable substance between phases; α_i, the mass transfer coefficient for particles with diameter d_i; p_i, the volume fraction of particles with diameter d_i; v, mean liquid velocity in a column; $Pe = vD/d_i$, Peclet number; D, the molecular diffusion coefficient; b_m, moments of the m-th-order size particle distribution; L, the reverse Laplace transformation; s, the parameter of the Laplace transformation carried out with respect to variable t_1; σ, the root-mean-square deviation-to-mean-random value ratio. Indices: in = initial conditions; i, index of the particle fraction with diameter d_i, $i = 1, 2, \ldots, N$.

LITERATURE CITED

1. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Moscow (1966).
2. Yu. A.Buevich, Inzh.-Fiz. Zh., **23**, No. 4, 709-712 (1972).
3. G. Dech, Manual to Practical Application of the Laplace Transformation and z-Transformation [in Russian], Moscow (1971).
4. A. Papoulis, Q. Applied Math., **14**, 405-414 (1957).